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MULTIDIMENSIONAL SCALING APPROACH TO THE DETERMINATION OF BASIC PSYCHOLOGICAL PARAMETERS FOR PURE TONES

ROBERT W. PETERS

MISSISSIPPI SOUTHERN COLLEGE

APRIL 1959

CONTRACT No. AF 33(616)-3644

AERO MEDICAL LABORATORY
WRIGHT AIR DEVELOPMENT CENTER
AIR RESEARCH AND DEVELOPMENT COMMAND
UNITED STATES AIR FORCE
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

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PROJECT NO. 7231
TASK NO. 71701

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WRIGHT AIR DEVELOPMENT CENTER
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FOREWORD

This report was initiated by the Aero Medical Laboratory, Wright Air Development Center, under Contract AF 33(616)-3644, in support of Project No. 7231, Task No. 71701, "Research on Psychological Parameters of Sound." Major Jack E. Steele, USAF (MC) of the Bioacoustics Branch served as contract monitor.

Basic research, compilation, and analysis of data were performed at Mississippi Southern College, Acoustic Laboratory, by Dr. Robert W. Peters, Project Director. Lt. H. Paul Kelley, USN, and Ens. Tony Morton, USN, of the U.S. Naval School of Aviation Medicine, Pensacola, Florida, gave advice on computational procedures. Maj. Steele was particularly helpful in his encouragement and served as liaison between the author and the Computer Branch, Aeronautical Research Laboratory, WADC, where part of the data were analyzed. These contributions are gratefully acknowledged.

ABSTRACT

The multidimensional scaling model of successive intervals was applied to investigate the dimensionality of auditory perception of pure tones. The stimuli consisted of 16 pure tones. Thirty-nine observers made distance judgments of similarity between stimuli. These inter-stimuli distances were analyzed mathematically to reveal the minimum number of dimensions necessary to account for the distances between stimuli. The results of the analysis indicated that there were two dimensions, pitch and loudness. The purpose of the study was to evaluate the multidimensional scaling method for use in auditory areas where the dimensions are not well known, and, since the two anticipated dimensions, pitch and loudness, were revealed, proposed use of the model in other auditory areas is supported.

PUBLICATION REVIEW

This report has been reviewed and is approved

FOR THE COMMANDER

Andres I. Karstens

ANDRES I. KARSTENS
Colonel, USAF (MC)
Asst. Chief, Aero Medical Laboratory

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INTRODUCTION

The purpose of this investigation was to apply a multidimensional scaling model to an auditory area of known dimensionality. The auditory area was the perception of pure tones and the model was the multidimensional method of successive intervals. The application of this model to a known perceptual area was done in order to evaluate multidimensional scaling for use in auditory areas where the dimensions are not well known.

Multidimensional scaling differs from traditional scaling methods in that judgments of similarity between stimuli may be utilized instead of judgments on a given continuum and, in that the dimensions and scale values are determined from the data instead of being specified by the experimenter (12). This approach has advantage in stimulus domains where the dimensions are unknown. In traditional methods the experimenter determines the dimensions on which the observer is to make his jugements. The observer needs to know what is meant by such terms as loudness, brightness, volume, and density. In some cases a dimension specified by the experimenter may be in reality complex and in other cases the dimension may not be relevant. In these situations the inadequacy of the traditional approach is apparent.

Multidimensional scaling has been applied in various fields to determine the nature and number of psychological dimensions. In the area of color perception, Richardson (7) and Messick (4) found good agreement between the results of multidimensional scaling and the Munsell color system (5). Attneave (1) differed size and shape as well as color and found that the scaling method revealed the appropriate number of dimensions. Klingberg (3) studied the mutual friendliness of seven great powers before World War II and noted for the most part that a three-dimensional system could account for mutual international distances. Messick (4) used the multidimensional method of successive intervals to evaluate attitudes toward war, capital punishment, and treatment of criminals. He found that the three attitudes could be represented in two dimensions, a war and punishment dimension.

Multidimensional scaling of the perception of pure tones would be expected to yield only pitch and loudness dimensions although other dimensions such as volume (8,11), brightness (2,9,10), and density (2,9,10) have been proposed. Osgood (6) suggests that brightness, density and volume may not be valid dimensions.

METHOD

The multidimensional scaling procedure used in this study involved three basic steps (4). First, comparative distances in similarity were obtained between all pairs of pure tone stimuli; second, an estimated

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additive constant was used to convert the comparative distances into absolute distances; and third, the dimensionality of the psychological space necessary to account for the absolute distances among the pure tone stimuli were determined.

Stimuli. The stimuli consisted of 16 pure tones, four frequencies at four sound pressure levels. The frequencies were 500, 1000, 2000, and 3000 cps each at the sound pressure levels of 70, 80, 90, and 100 db (re .0002 dyne/cm^2)(See Table I).

Table I.

Pure Tone Stimuli Identified by Stimulus Number

Stimulus Number	Frequency (cps)	Sound Pressure Level (db)
1	500	70
2	1000	70
3	2000	70
4	3000	70
5	500	80
6	1000	80
7	2000	80
8	3000	80
9	500	90
10	1000	90
11	2000	90
12	3000	90
13	500	100
14	1000	100
15	2000	100
16	3000	100

Apparatus. The experimental apparatus included a pure tone oscillator, General Radio, Model 1302-A, two magnetic tape recorders, Ampex, Models 350 and 350-3, a condenser microphone, Altec-Lansing, Model 21-D, an attenuator, Hewlitt-Packard, Model 350, an audio console, Altec-Lansing, Model 1560A, a voltmeter, Hewlitt-Packard, Model 400AB, and a headset circuit with PDR-3 receivers.

Observers. Thirty-nine male and female college students served as observers.

Procedure. The 16 pure tones were programmed in all possible pair combinations. This resulted in 120 pairs (n(n-1)/2). The duration of each tone was one second. A one-second interval separated the two tones of each pair. An identifying carrier number preceded each stimulus pair with an interval of seven and a half seconds between the onset of each carrier number. The presentation order of the pairs of tones was determined by random selection.

The observers' task was to judge each pair of sounds for similarity on a nine point scale. Number <u>one</u> on the scale represented extreme similarity and number <u>nine</u> represented extreme dissimilarity. The directions for making the judgments were tape recorded and also printed on the first page of the test booklet so that the observers both heard and read the directions prior to each experimental session (See Appendix A). Twenty practice pairs of tones preceded the test stimuli. After the observers had judged the practice stimuli, the directions were briefly repeated.

The observers heard the stimuli through headsets and took part in the experiment in groups of nine or less. The comparative distances between stimuli were determined on the basis of the observer's judgments.

ANALYSIS AND RESULTS

The raw data were tabulated in a 120 (stimulus pairs) x 9 (scale categories) table where the cell values, f_{ig}, indicated the frequency that the i^{th} pair of stimuli were placed in the g^{th} category (See Table II, Appendix B). The frequencies were cumulated, converted to proportions, and from the proportions were determined normal deviate values, z_{ig}. The deviate values were weighted according to the function, Z^2/pq, where Z was the ordinate of the normal curve which corresponded to the proportion, p, and $q = (1-p)$. A table of successive differences in deviate values was constructed and the differences themselves were weighted according to the formula, $W_1W_2/W1+W_2$, where W_1 referred to the weight applied to one deviate value and W_2 referred to the weight applied to the other deviate value. Weighted averages of successive differences were determined and scale values, t_g, were computed. This scale was used as the ordinate in determining graphically the scale values and discriminal dispersions for each stimulus pair. The resulting scale values for each pair were converted to positive values by setting the smallest scale value to zero. These values, s_{jk}, represented comparative interpoint distances and are shown in Table III, Appendix B.

Matrices, S^2_{jk}, squared relative distances, A, E, and H were constructed. The elements, a_{jk}, e_{jk}, and h_{jk} of matrices A, E, and H were:

$$a_{jK} = \frac{1}{2}\left(\frac{1}{n}\sum_{K}^{n} S^2_{jK} + \frac{1}{n}\sum_{j}^{n} S^2_{jK} - S^2_{jK} - \frac{1}{n^2}\sum_{jK}^{nn}\sum S^2_{jK}\right)$$

$$e_{jK} = \frac{1}{n}\sum_{K}^{n} S_{jK} + \frac{1}{n}\sum_{j}^{n} S_{jK} - S_{jK} - \frac{1}{n^2}\sum_{jK}^{nn}\sum S_{jK}$$

$$h_{jK} = \frac{1}{n} \text{ for } j \neq K \text{ and } h_{jj} = \left(1 - \frac{1}{n}\right)$$

A B* matrix was determined ($B^* = A + cE + 1/2c^2H$) using as the additive constant, c = 3.75 (See Table IV, Appendix B).

The B* matrix was solved for eigen values and eigen vectors.* Five non-zero roots and their corresponding vectors were retained and the coeficients for the sum of the latent roots of the B* matrix were computed using the following equation:

$$\sum \beta = X_i' A X_i + c_i^p X_i' E X_i + 1/2\ pc^2$$

In the above equation p = the number of roots and c is unknown. The coeficients for the sum of the diagonals of the B* matrix were determined. The equation for the sum of the diagonal elements was:

$$\sum_j^n b^*_{jj} = \tfrac{1}{2}n \sum_{jK}^{nn} s^2_{jK} + c/n \sum_{jK}^{nn} s_{jK} + \tfrac{1}{2}(n-1)c^2$$

The two resulting equations were:

$$\sum \beta = 44.6125 + 14.7534\ c + 2.5\ c^2$$

$$\sum_j^n b^*_{jj} = 35.7818 + 29.6663 + 7.5\ c^2$$

These equations were set to equal each other and the resulting quadratic was solved for two values of c: $c = -5.8147$ and $c = 0.8437$. The root which gave the largest $\Sigma\beta$ was $c = 0.8437$. Because c was a small positive number, the analysis was continued with c set for $c = 0$.

A factor matrix was computed using the general matrix factoring solution, $(BX)K^{-1} = F$, where X is the matrix of X_i vectors and K is the matrix where $X'BX = K'K$. The factor matrix is shown in Table V, Appendix B. The projections of the stimuli on the axes for the five factors by pairs are shown in Figures 1-10, Appendix C.

Orthogonal rotation of factors was then done to achieve simple structure and meaningful dimensions. The orthogonal transformation matrix is shown in Table VI, Appendix B, and the final rotated matrix is shown in Table VII of Appendix B. The projections of stimuli for the final rotated matrix are shown in Figures 11-20, Appendix C.

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*These computations were made at the Computer Branch, Aeronautical Research Laboratory, Wright Air Development Center.

DISCUSSION

The results would seem to indicate that two factors, pitch and loudness, are fairly well defined. A third factor, not as well defined, seems to relate to pitch. Factor I appears to be a pitch dimension with the frequencies of 500, 1000, and 2000 cps producing a continuum for the several sound pressure levels. The exception for this factor is that the 3000 cps stimuli occupy approximately the same scale positions as the 2000 cps tones. The major loadings for factor II are accounted for by the four 3000 cps stimuli and with respect to these stimuli appears to relate to pitch. Factors III and IV do not have sufficient loadings to be considered legitimate dimensions. Factor V appears to be a loudness dimension which is best defined by the scale positions for the 1000, 2000, and 3000 cps stimuli and not well defined for the 500 cps stimuli.

It would appear on the basis of the obtained results that the multidimensional scaling model was successful in isolating the basic psychological parameters, pitch and loudness, for pure tones and could therefore be of value as a model for exploring auditory areas where the dimensions are not well known.

SUMMARY AND CONCLUSIONS

The multidimensional scaling method of successive intervals was used to evaluate the perception of pure tones. Sixteen pure tones comprised the stimuli which were judged for similarity by 39 observers. The results indicate that two factors, loudness and pitch, are well defined and suggest that the multidimensional model would be of value in investigating the dimensionality of auditory areas were the dimensions are not known.

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APPENDIX A

DIRECTIONS TO OBSERVERS

You are asked to judge pairs of sounds for similarity. You are to judge each pair of sounds in relation to a nine point scale.

The scale is one of equal steps with 1 representing extreme similarity and 9 representing extreme dissimilarity. Step 5 is thus halfway between 1 and 9 with the other points falling on the scale equal distances apart.

You are to use all of the nine points in making your judgments. Indicate your judgment of similarity by circling the number on the scale which corresponds to your choice. Do not skip any of the pairs in making your judgments. Make a judgment with respect to each pair that you hear.

The first 20 pairs of sounds that you hear are to be judged for practice and to acquaint you with the range of similarity among the pairs of sounds.

Here are the practice pairs of sounds.

Now turn the page and be prepared to judge the following pairs of sounds. Remember that 1 on the scale represents extreme similarity and number 9 represents extreme dissimilarity.

APPENDIX B

Table II.

F_{ig}, The Frequency With Which The *i*th Pair of Stimuli

Was Placed In The *g*th Category

Stimulus Pair	Category 1	2	3	4	5	6	7	8	9
5-11				3	5	2	15	7	7
12-7		4	4	6	11	10	2	1	1
13-10		1	2	8	8	7	7	6	
8-15		3	4	3	11	5	9	2	2
10-16			1	2	4	8	9	12	3
9-2		6	5	10	2	6	5	4	1
4-6	1	4	3	7	7	6	7	3	1
15-16	2	4	7	8	6	3	5	2	2
1-11			3	5	6	15	4	2	4
16-7			8	4	12	8	6	1	
9-10		2	9	8	4	9	5	2	
4-1		1	3	5	4	4	5	8	9
2-12		2		1	4	7	11	8	6
15-6	1	3	4	6	8	6	6	4	1
7-11	26	11	2						
12-10		2	2	4	5	8	11	5	2
6-14	13	13	6	3				3	1
4-8	32	5	1	1					
16-13			2	1		2	3	10	21
1-9	17	15	4		1	1		1	
9-3				2	8	9	7	8	5
4-11		1	8	15	4	9	1	1	
5-7			8	3	7	10	8	3	
2-4	1	1	3	7	10	8	5	4	
3-13	1		1		3	3	6	10	15
10-7		4	9	4	6	8	4	4	
3-6			6	7	10	7	5	4	
6-2	23	12	2	2					
10-14	17	12	4	2		1	1	1	1
11-2	1	2	5	8	6	6	6	4	1
1-2		4	8	9	7	9	1	1	
3-15	8	15	2		3	2	1	5	3
2-16			1	2	2	4	9	11	10
5-15	1			1	2	10	7	12	6
12-4	14	16	3	2	3			1	
1-6		2	6	11	8	7	5		
11-3	12	16	7	2		1		1	
9-14		1	3	12	9	4	7	3	
8-6		1	1	4	6	13	11	2	1
14-12				1	5	7	8	10	8

APPENDIX B (cont.)

Table II.

Stimulus Pair	Category 1	2	3	4	5	6	7	8	9
8-5		1		1	6	4	11	7	9
15-9			2	2	1	3	9	13	9
9-5	16	18	2	2		1			
3-7	30	7			1	1			
10-6	17	19	2		1				
12-6		1		6	8	7	11	4	2
8-12	19	9	5	3	2	1			
9-12				1	4	6	9	11	8
10-5	1		3	4	6	6	10	5	4
2-7			1	6	12	9	5	5	1
7-14			3	7	2	5	7	6	9
11-15	17	11	2	3	5		1		
12-15	1	3	8	11	8	3	4		1
13-5	9	17	8	2		1		1	1
1-8	1		1		7	6	7	11	6
1-14		1	4	2	2	7	9	5	9
5-3			2	3	11	10	7	4	2
13-7			3	2	5	4	7	11	7
8-14		1		1	4	9	8	9	7
10-8			1	3	8	7	9	8	3
4-9				3	4	4	8	14	6
2-14	13	8	8	4	1	3		1	1
3-16	2	3	5	4	4	5	7	4	5
6-16	1		1	4	2	6	7	12	6
3-1		1	3	1	5	7	10	6	6
12-1	1		2	1	3	4	7	10	11
10-4	2	1	1	5	4	8	12	4	2
13-6		1	3	7	12	6	2	5	3
12-5		2	2	5	4	4	6	10	6
3-2	3	2	10	9	4	6	2	3	
7-15	14	15	7	1	2				
16-9	1			1	3	7	7	9	11
12-11		8	5	10	7	4	1	2	2
14-15	2	2	3	10	6	6	7	3	
14-13	1		5	2	4	9	10	2	6
9-13	33	4	2						
4-5	1	1	6	4	7	8	7	4	1
6-5		3	5	5	7	6	10	2	1
9-8	1		1	3	4	5	2	16	7
11-6	2	[illegible]		8	7	13	1	6	
1-10		1	6	5	9	15	1	1	1
13-2		1	7	7	6	7	4	3	4
2-5	2	2	8	10	7	7	3		
7-9	1	1	2	3	8	12	5	6	1

APPENDIX B (cont.)

Table II.

Stimulus Pair	Category 1	2	3	4	5	6	7	8	9
4-3	2	8	9	10	4	3	1	1	1
14-16		1	1	4	4	10	7	6	6
13-11			3	3	6	10	11	3	3
8-11	1	7	7	10	2	3	3	1	
6-9	1	2	3	8	9	5	5	5	1
14-3	1	2	8	6	5	8	3	4	2
1-5	29	8		1			1		
10-2	15	16	4	2		1	1		
16-8	16	11	8	2			1	1	
12-16	27	5	2	4	1				
13-1	10	17	4	3	1	1		2	1
14-11		3	8	10	12	3		3	
3-12	1		6	10	9	7	3	2	1
16-1		1			4	6	10	12	6
1-7			3	4	6	9	8	7	2
2-8	2			4	10	10	6	5	2
15-10	1		4	6	15	8	4	1	
11-16	2	1	1	8	7	6	4	7	3
15-13		1		1	5	1	8	11	12
15-1	1	1		3	1	1	12	11	9
11-9		1	1	4	3	2	13	11	4
13-8			1		2	7	8	8	13
15-2	1	1	3	4	7	8	6	8	1
3-10	1	1	12	7	6	7	4	1	
11-10	1	1	9	9	6	8	4	1	
4-16	14	11	4	3	1	3	2		1
4-13			1		3	5	4	12	14
16-5	1			3	3	7	5	14	6
7-4		4	11	7	5	6	3	1	2
6-7	2	4	9	11	7	5	1		
14-4	1		1	5	8	9	7	5	3
7-8	3	9	9	5	5	6	1	1	
13-12			1	1	5	3	8	9	12
5-14			5	6	6	4	9	7	2
15-4	2	2	6	8	5	6	3	4	3
3-8	3	5	11	7	6	3	1	2	1

APPENDIX B (cont.)

Table III.

s_{jk}, Comparative Interpoint Distances for the 16 Pure Tone Stimuli

Items	1	2	3	4	5	6	7	8
1	0.0000	2.9929	7.4529	7.8400	0.0016	3.5344	6.1009	8.2944
2		0.0000	2.7225	4.4100	3.0276	0.0841	4.8400	5.4756
3			0.0000	1.9881	5.4756	4.3264	0.0000	2.0736
4				0.0000	3.2761	4.2025	2.7556	0.0000
5					0.0000	4.7089	4.8400	8.7025
6						0.0000	2.4336	5.8081
7							0.0000	3.9204
8								0.0000

Items	9	10	11	12	13	14	15	16
1	0.2401	4.3264	5.0625	9.6721	0.5476	7.6729	9.3636	8.5849
2	3.8025	0.3025	4.1209	8.1225	4.3681	0.6889	5.6169	9.6100
3	7.2361	2.8900	0.4624	3.6864	11.2225	3.7636	0.7225	5.2441
4	9.3025	11.2896	3.1684	0.3600	10.8241	5.7600	4.0000	0.4900
5	0.2500	6.0516	8.3521	7.7841	0.6241	5.6644	8.4681	8.2369
6	4.4521	0.2116	4.4100	5.7121	4.7961	0.4900	3.8025	8.1225
7	5.3361	3.7249	0.0529	2.0164	8.5264	12.1758	0.3721	3.9204
8	7.9524	6.9696	2.4336	0.1849	9.7344	8.2369	4.4944	0.3249
9	0.0000	3.4969	8.2944	9.0000	0.0000	3.8416	9.7969	9.3025
10		0.0000	3.2400	6.2500	5.0625	0.2809	4.0401	7.9524
11			0.0000	2.6569	6.2001	2.8224	0.2916	4.9729
12				0.0000	9.7969	8.7025	2.8900	0.0000
13					0.0000	6.2500	1.4641	14.0625
14						0.0000	3.8809	7.1289
15							0.0000	3.1329
16								0.0000

APPENDIX B (cont.)

Table IV.

B* Matrix for c = 3.75

Items	1	2	3	4	5	6
1	13.347500	0.546453	- 3.675147	- 3.447593	5.542215	- 0.175949
2		11.160802	- 0.442048	- 1.577194	0.062715	3.373368
3			10.927602	0.810906	- 2.280987	- 3.628944
4				12.032309	0.364867	- 1.937057
5					12.823524	- 1.632689
6						10.819997

Items	7	8	9	10	11	12
1	- 2.558941	- 3.539208	5.406642	- 0.458453	- 1.700278	- 4.555654
2	- 2.579342	- 2.274409	- 0.269159	3.265146	- 1.974079	- 4.450456
3	3.849058	0.997491	- 3.490059	- 0.301454	2.269821	- 0.605256
4	- 0.088888	5.286645	- 4.645906	- 7.111401	- 0.593326	4.085298
5	- 1.747981	- 4.200248	5.057202	- 2.359293	- 4.870818	- 3.337394
6	- 0.346544	- 2.742312	- 1.064362	3.308943	- 2.420282	- 2.553158
7	10.847527	- 0.985703	- 3.684228	- 1.197448	3.271027	1.119951
8		12.603479	- 3.254021	- 3.265816	0.472160	4.777183
9			13.478379	0.351735	- 4.495791	- 4.011667
10				11.796989	- 0.784286	- 2.539862
11					10.686939	0.332913
12						12.810787

Items	13	14	15	16
1	5.368928	- 2.848243	- 5.020881	- 2.858897
2	0.113977	3.124156	- 3.010882	- 4.847298
3	5.257067	- 0.611044	2.169718	- 1.261998
4	- 5.271919	- 1.919391	- 1.072928	4.648456
5	4.911188	- 1.303483	- 4.617621	- 2.898137
6	- 0.801576	3.296953	- 1.486584	- 3.805201
7	- 4.028967	- 6.345688	2.747625	- 0.066392
8	- 4.122734	- 3.753506	- 1.259543	5.260341
9	7.031916	0.587863	- 5.367093	- 3.441009
10	- 0.558779	4.208750	- 1.229388	- 3.175404
11	- 2.132604	0.226725	2.846087	- 1.134429
12	- 4.069081	- 4.032652	0.433811	6.595195
13	14.647952	- 1.044069	2.984093	- 6.548723
14		12.951409	- 0.497578	- 1.905138
15			11.384334	0.996768
16				14.442102

APPENDIX B (cont.)

Table V.

Matrix of Projections

Items	I	II	III	IV	V
1	- 1.254168	0.928266	0.022805	0.457372	- 0.222037
2	- 0.855545	- 0.513561	- 0.454614	0.060551	0.842415
3	0.314215	- 0.407297	1.172586	- 0.687206	0.536645
4	1.299043	0.708430	- 0.353618	- 0.290386	1.048440
5	- 1.121258	1.236771	- 0.012506	- 0.177327	0.508590
6	- 0.548804	- 0.765153	- 0.504127	0.379228	0.372046
7	0.745503	0.096135	0.862530	1.322291	0.036674
8	1.373313	0.426406	- 0.195914	- 0.421182	0.026929
9	- 1.551927	0.846110	- 0.148172	- 0.055706	- 0.509485
10	- 0.797413	- 1.208411	- 0.204264	0.304264	- 0.679314
11	0.662972	- 0.707179	0.614065	0.148692	0.026100
12	1.452643	0.391705	- 0.002657	- 0.118147	- 0.391474
13	- 1.595211	0.357876	1.540215	- 0.791996	0.018424
14	- 0.712016	- 1.122525	- 0.876100	- 1.299269	- 0.048897
15	0.704506	- 0.776790	1.115610	- 0.485412	- 0.258071
16	1.646106	0.620197	- 0.505881	- 0.400257	- 0.628188

Table VI.

Orthogonal Transformation Matrix

Items	I	II	III	IV	V
1	0.707100	- 0.707100	0.000000	0.000000	0.000000
2	0.593832	0.593832	- 0.250594	- 0.3.9694	0.357600
3	0.050281	0.050281	0.929555	- 0.239771	0.268200
4	0.282570	0.282570	0.199809	0.894000	0.000000
5	0.252617	0.252617	0.178629	- 0.199809	- 0.894000

APPENDIX B (cont.)

Table VII.

Final Rotated Matrix of Projections

Items	I	II	III	IV	V
1	- 1.543199	- 0.424865	- 0.164403	0.321357	0.028859
2	- 0.241817	- 0.417206	- 0.280011	- 0.423572	- 1.192284
3	0.510181	0.062482	1.394003	- 0.406371	- 0.156507
4	0.417622	1.748473	0.123048	0.236991	- 0.435328
5	- 1.667362	0.310291	0.173105	- 0.128389	- 0.392301
6	0.152980	- 0.642132	- 0.525826	- 0.132984	- 0.830362
7	0.459168	- 0.125967	0.536876	1.592291	0.069693
8	0.669558	1.262105	0.016588	0.092865	0.479725
9	- 1.695652	- 0.546389	- 0.296510	- 0.278850	0.261841
10	0.290617	- 1.479993	- 0.546379	- 0.335696	0.003222
11	0.968834	- 0.218335	0.539932	0.243135	0.045479
12	0.750189	0.993678	0.013601	0.415003	0.839024
13	- 1.381028	- 0.860953	1.564339	- 0.749929	0.104332
14	0.290271	- 0.471981	- 0.608213	- 1.854985	- 0.316614
15	1.047424	- 0.259593	1.080559	- 0.231475	0.508725
16	0.725420	1.375894	- 0.428802	0.181480	1.123717

APPENDIX C

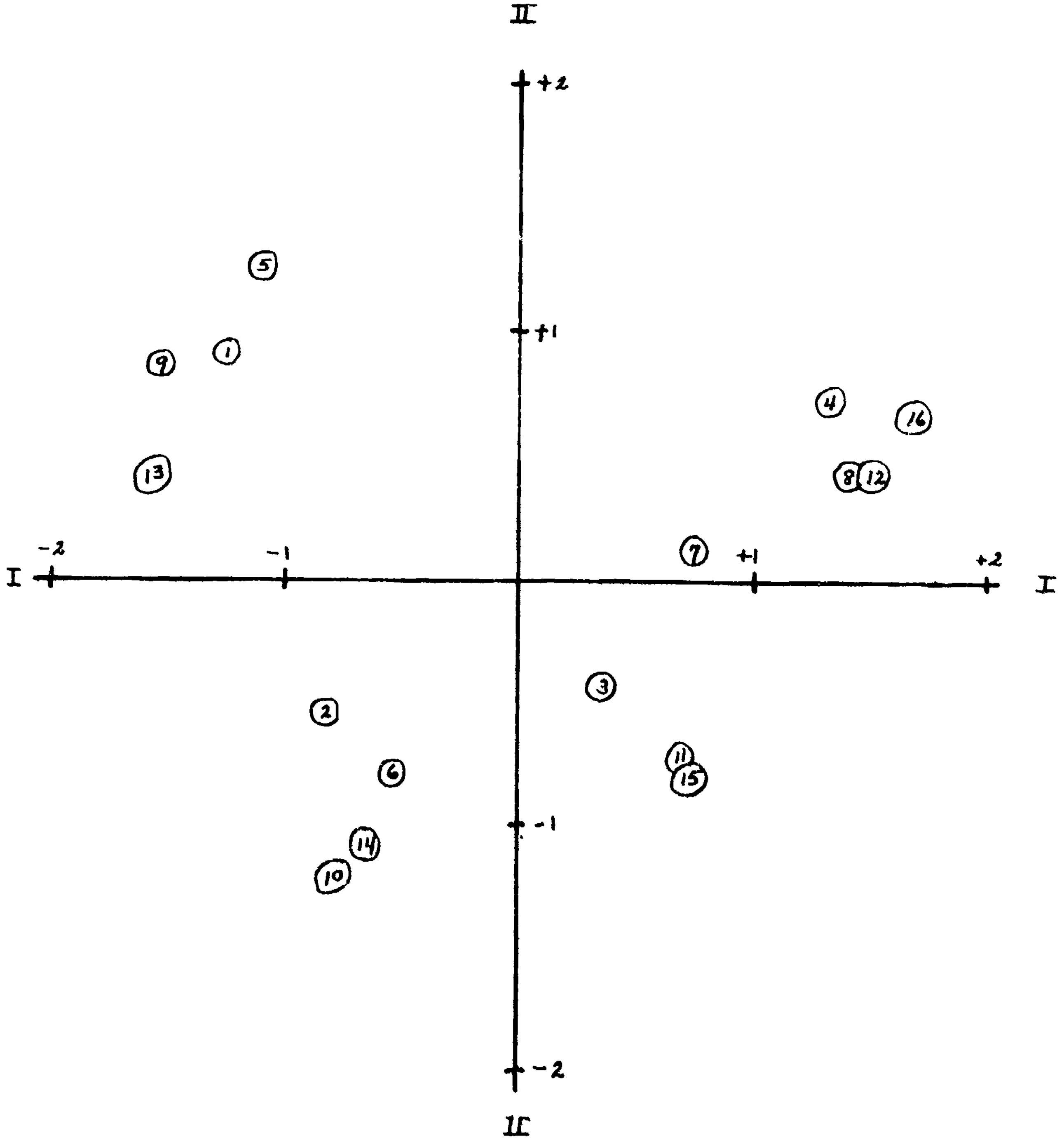

Figure 1. Factor I versus Factor II, Unrotated Factor Matrix, for 16 Pure Tone Stimuli.

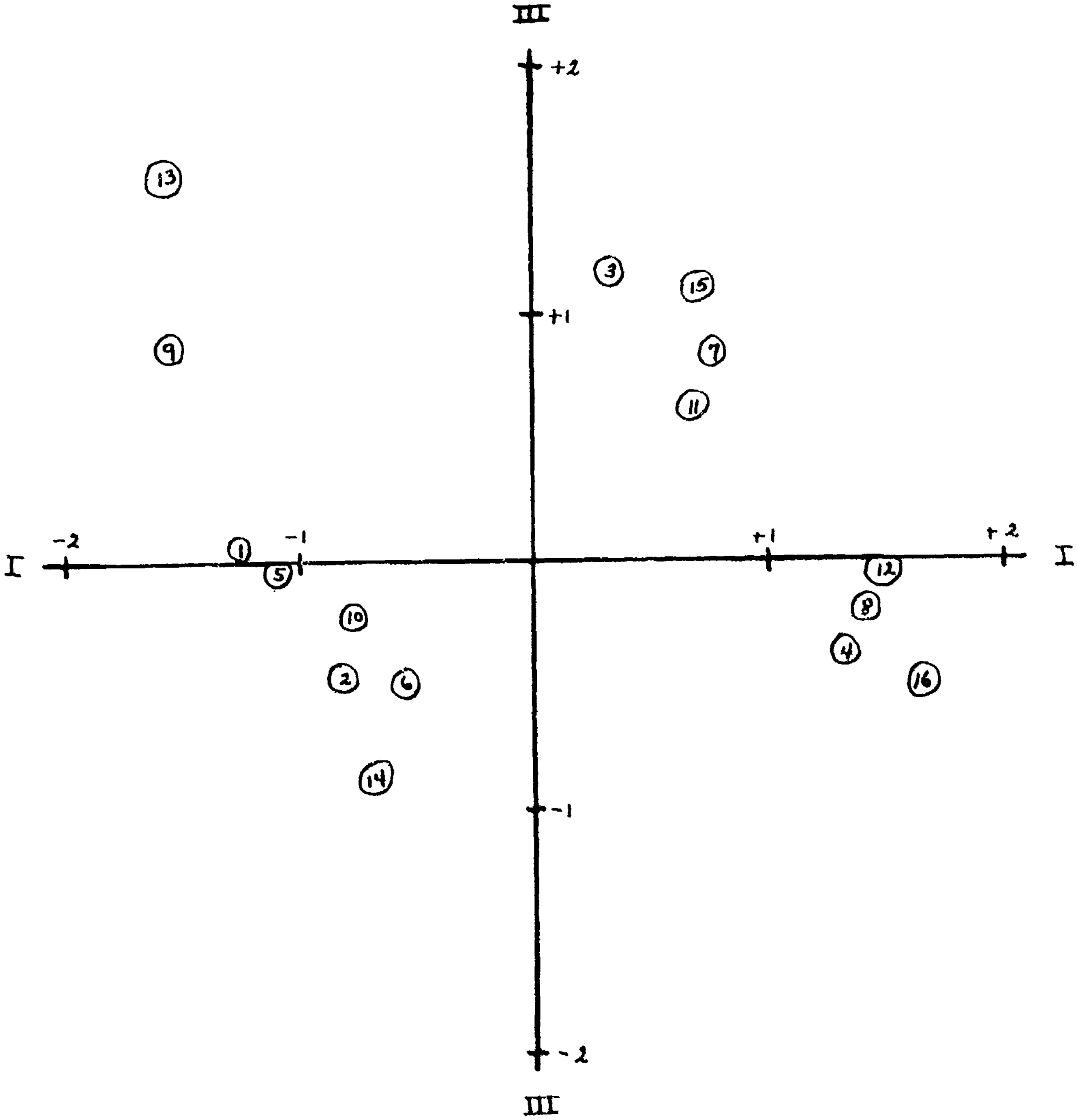

Figure 2. Factor I versus Factor III, Unrotated Factor Matrix, for 16 Pure Tone Stimuli.

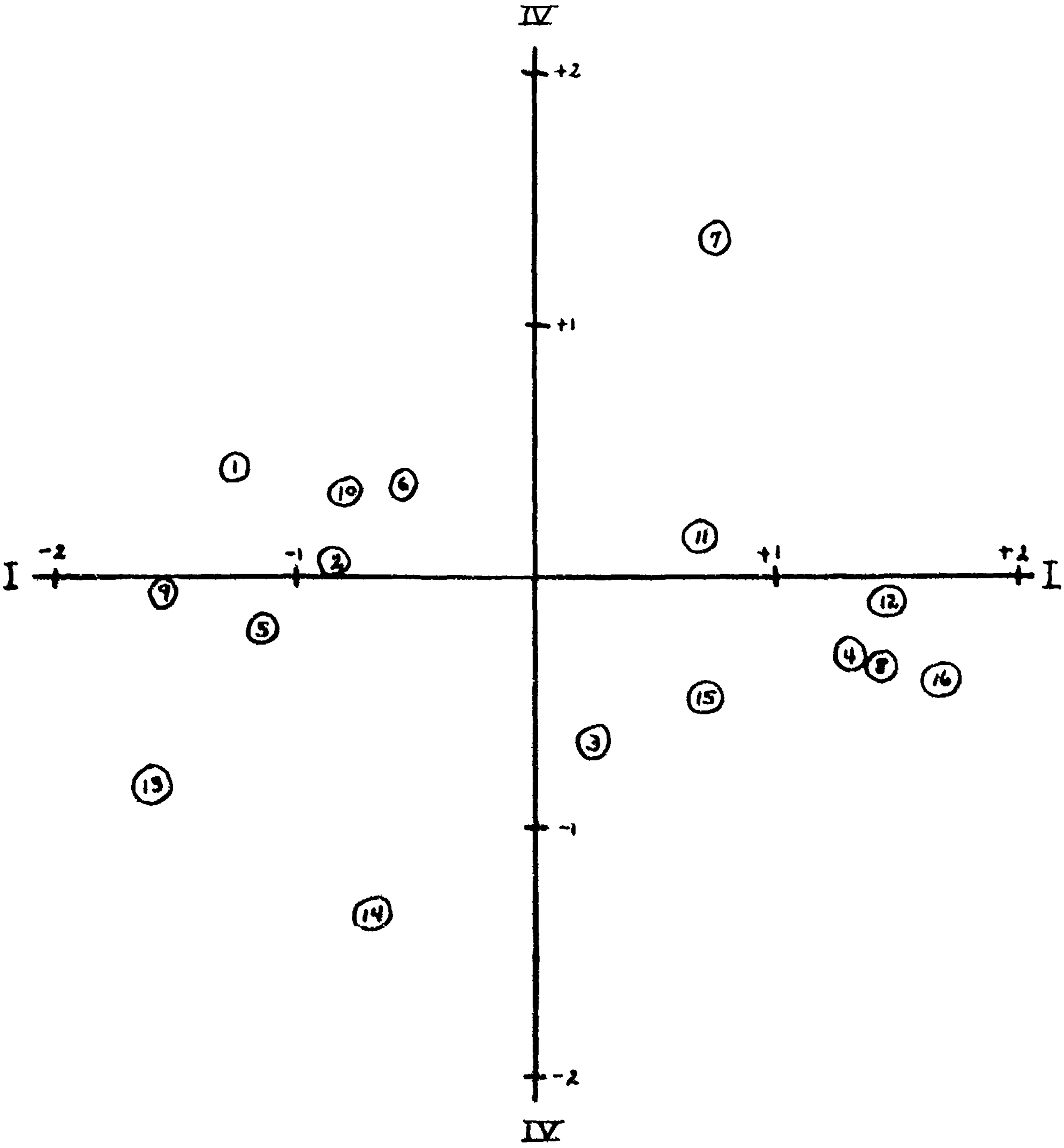

Figure 3. Factor I versus Factor IV, Unrotated Factor Matrix, for 16 Pure Tone Stimuli.

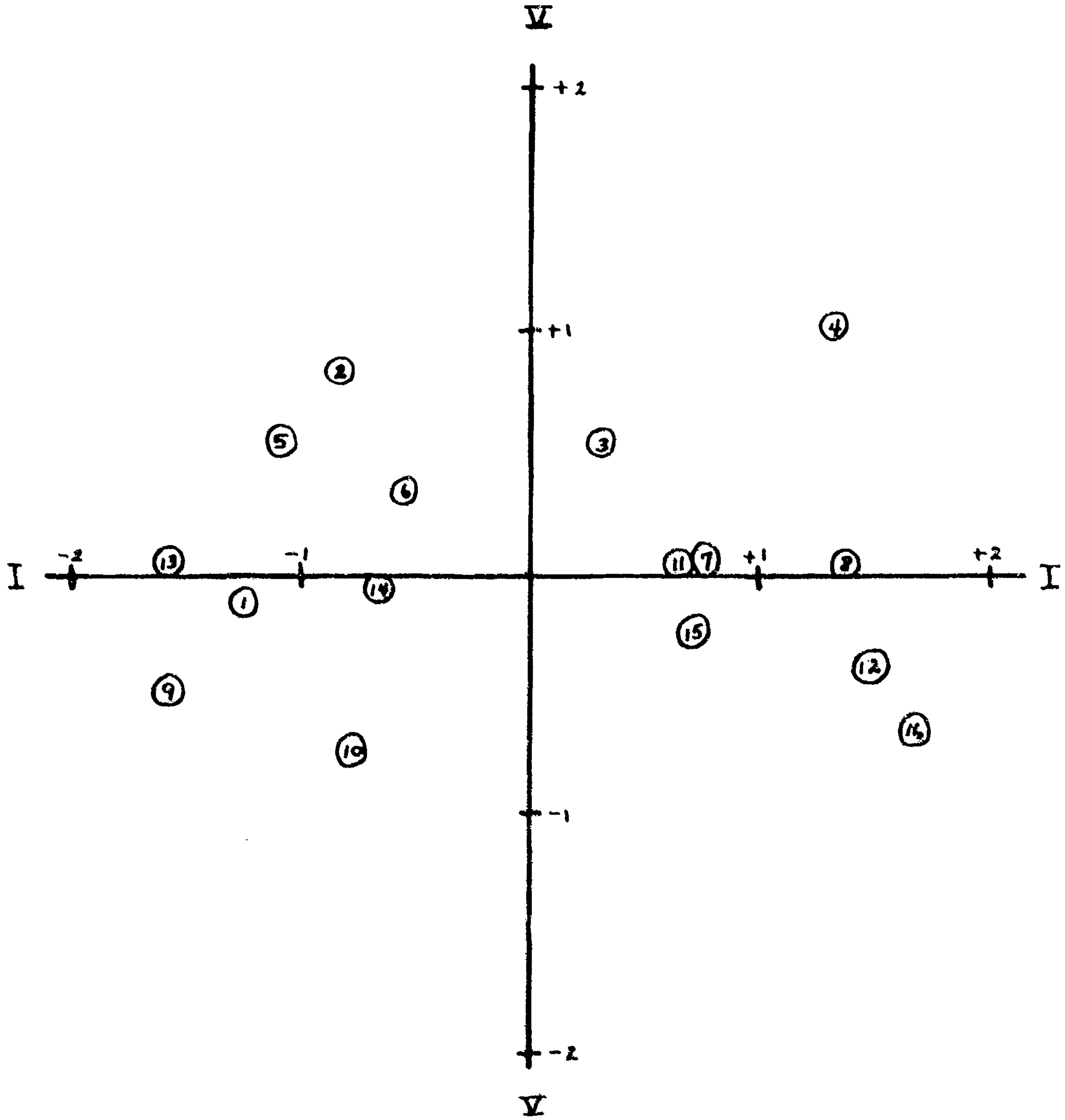

Figure 4. Factor I versus Factor V, Unrotated Factor Matrix, for 16 Pure Tone Stimuli.

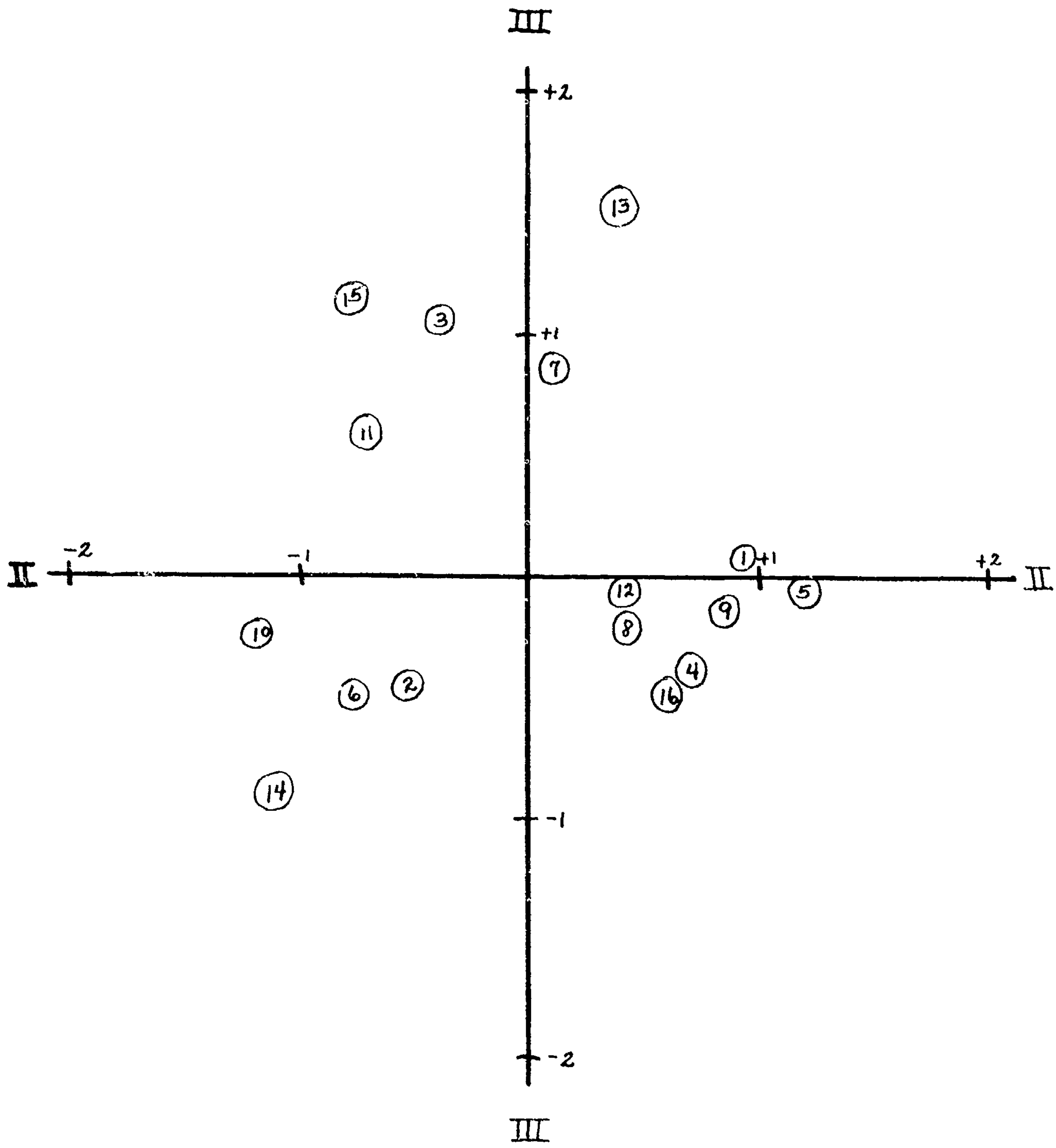

Figure 5. Factor II versus Factor III, Unrotated Factor Matrix, for 16 Pure Tone Stimuli.

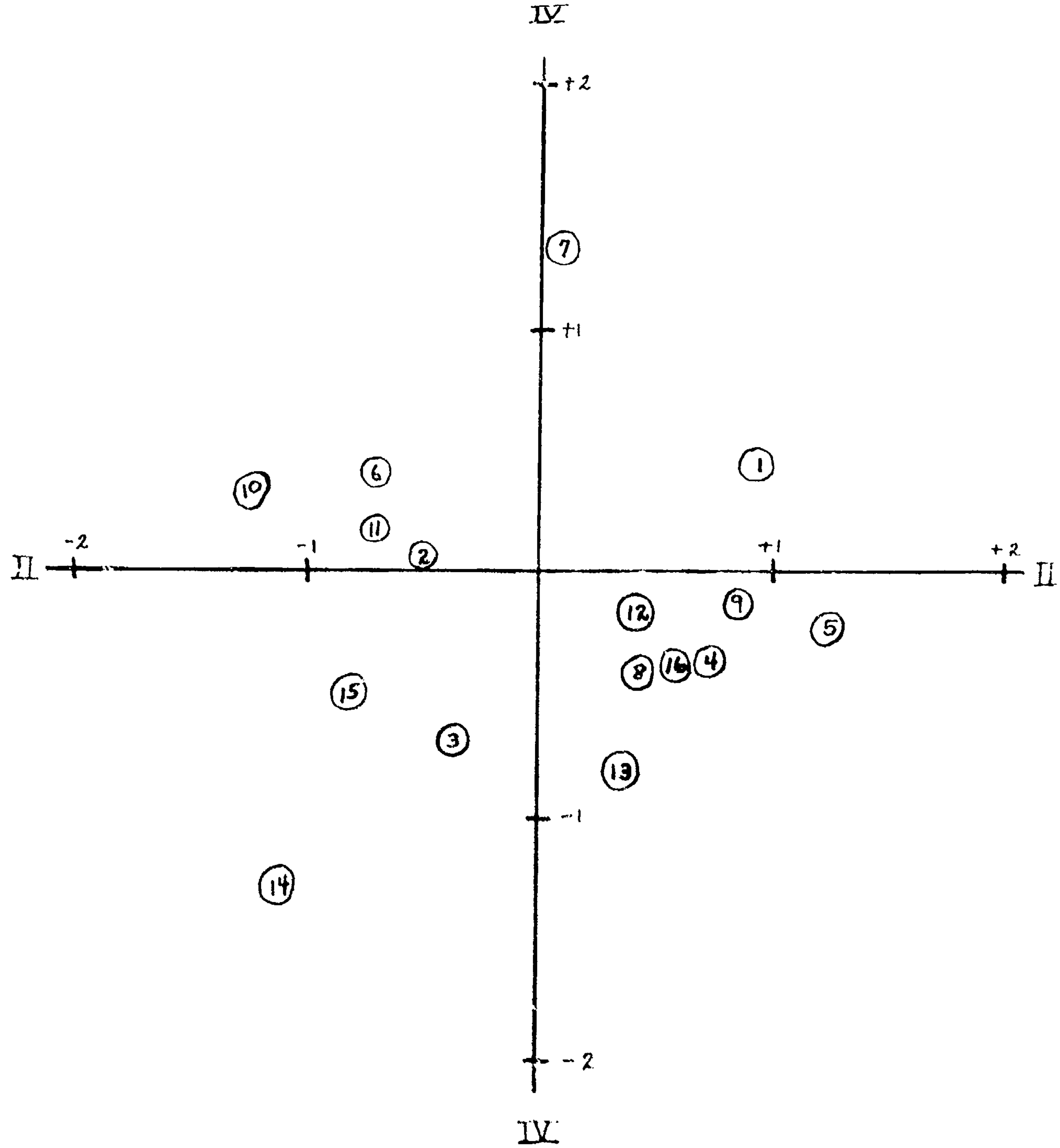

Figure 6. Factor II versus Factor IV, Unrotated Factor Matrix, for 16 Pure Tone Stimuli.

APPENDIX C (cont.)

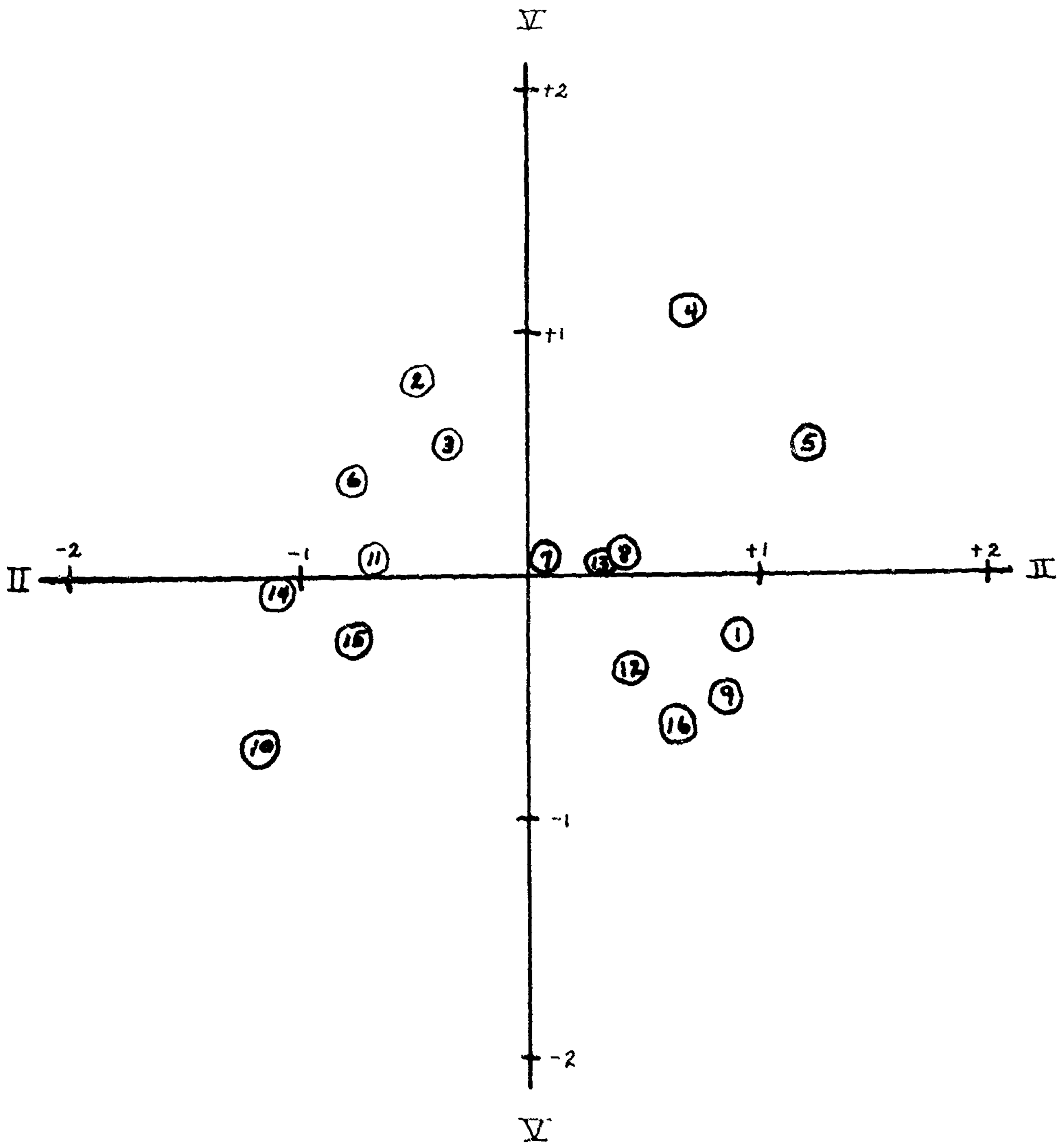

Figure 7. Factor II versus Factor V, Unrotated Factor Matrix, for 16 Pure Tone Stimuli.

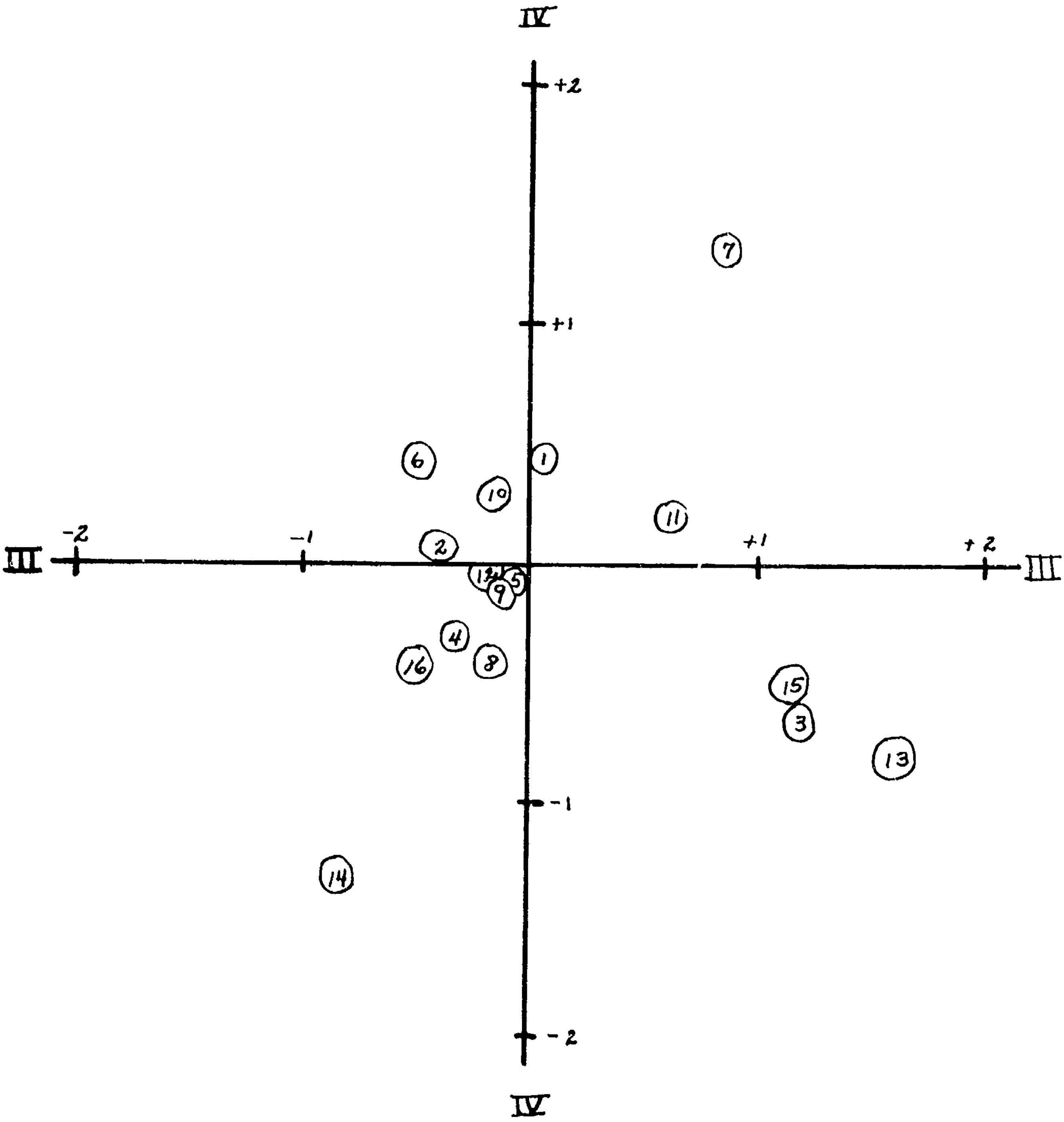

Figure 8. Factor III versus Factor IV, Unrotated Factor Matrix, for 16 Pure Tone Stimuli.

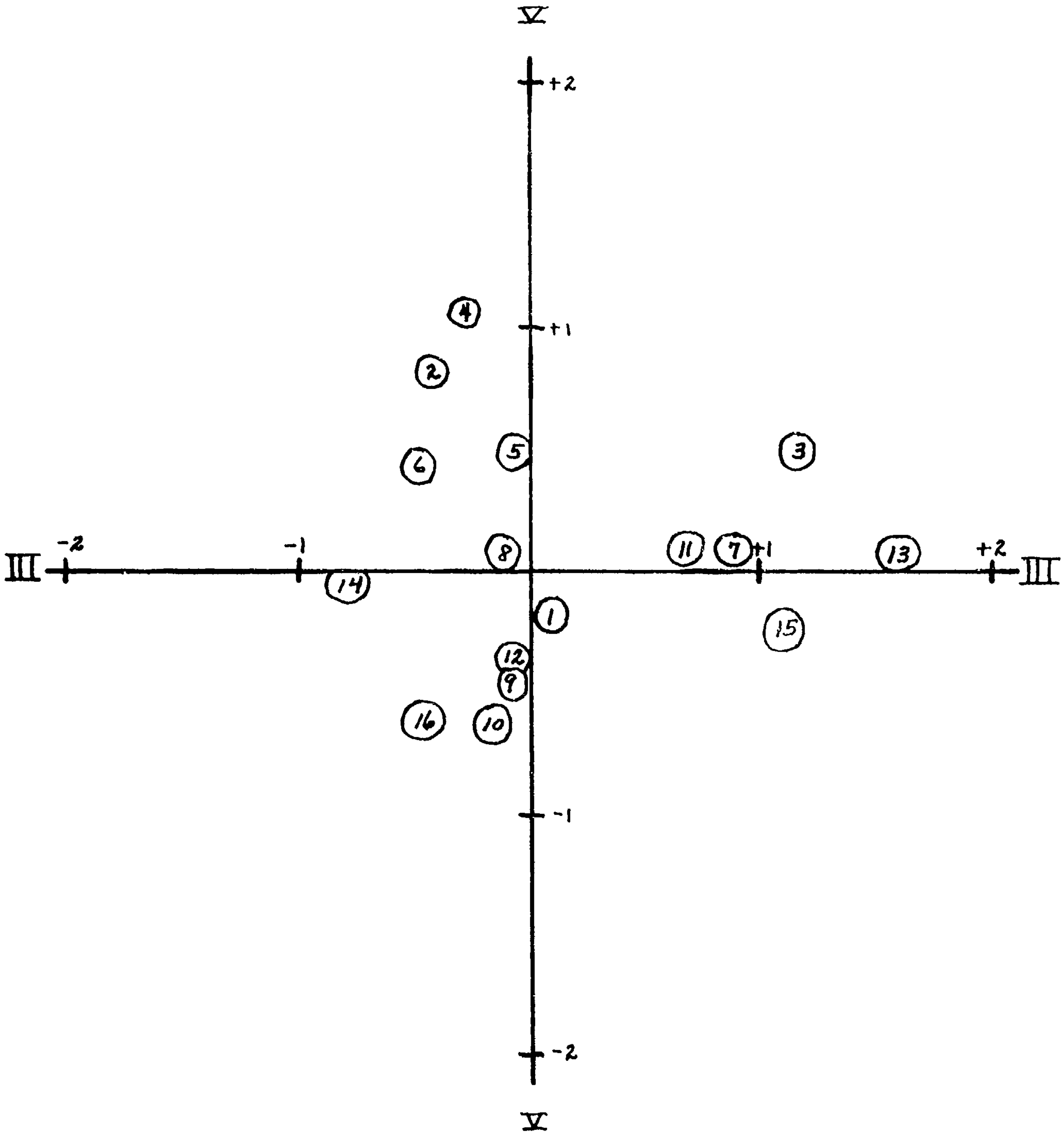

Figure 9. Factor III versus Factor V, Unrotated Factor Matrix, for 16 Pure Tone Stimuli.

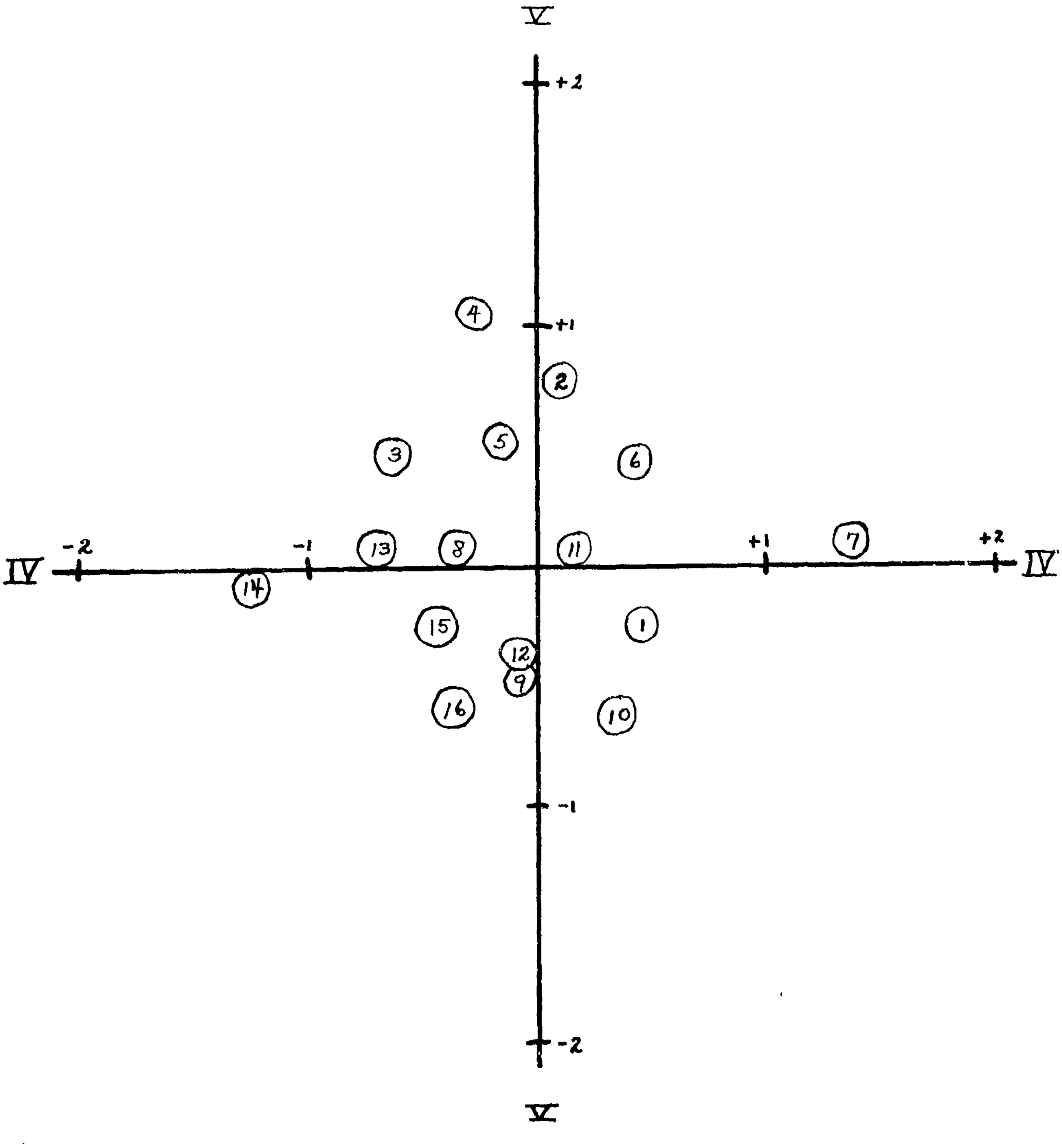

Figure 10. Factor IV versus Factor V, Unrotated Factor Matrix, for 16 Pure Tone Stimuli.

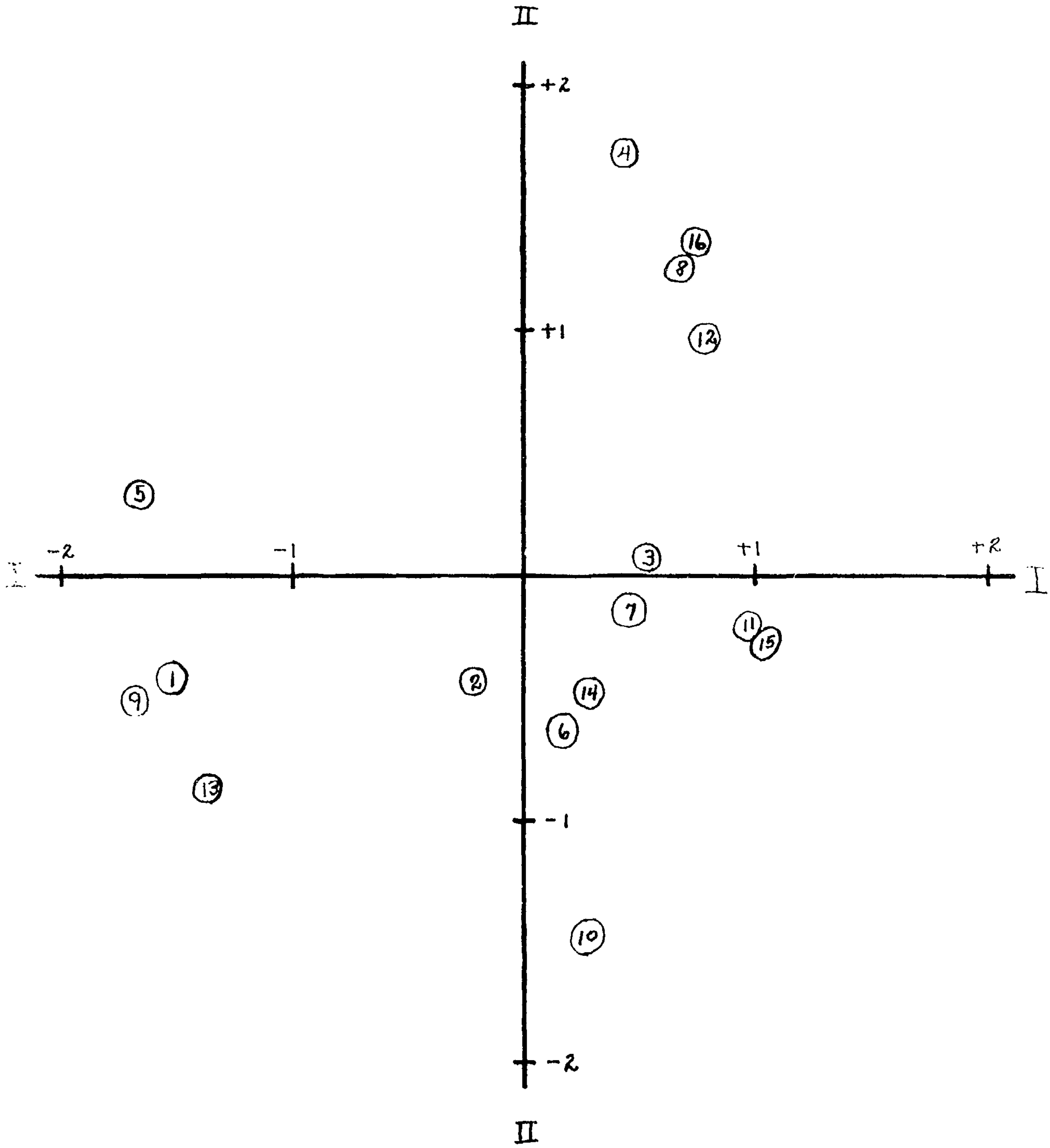

Figure 11. Factor I versus Factor II, Rotated Factor Matrix, for 16 Pure Tone Stimuli.

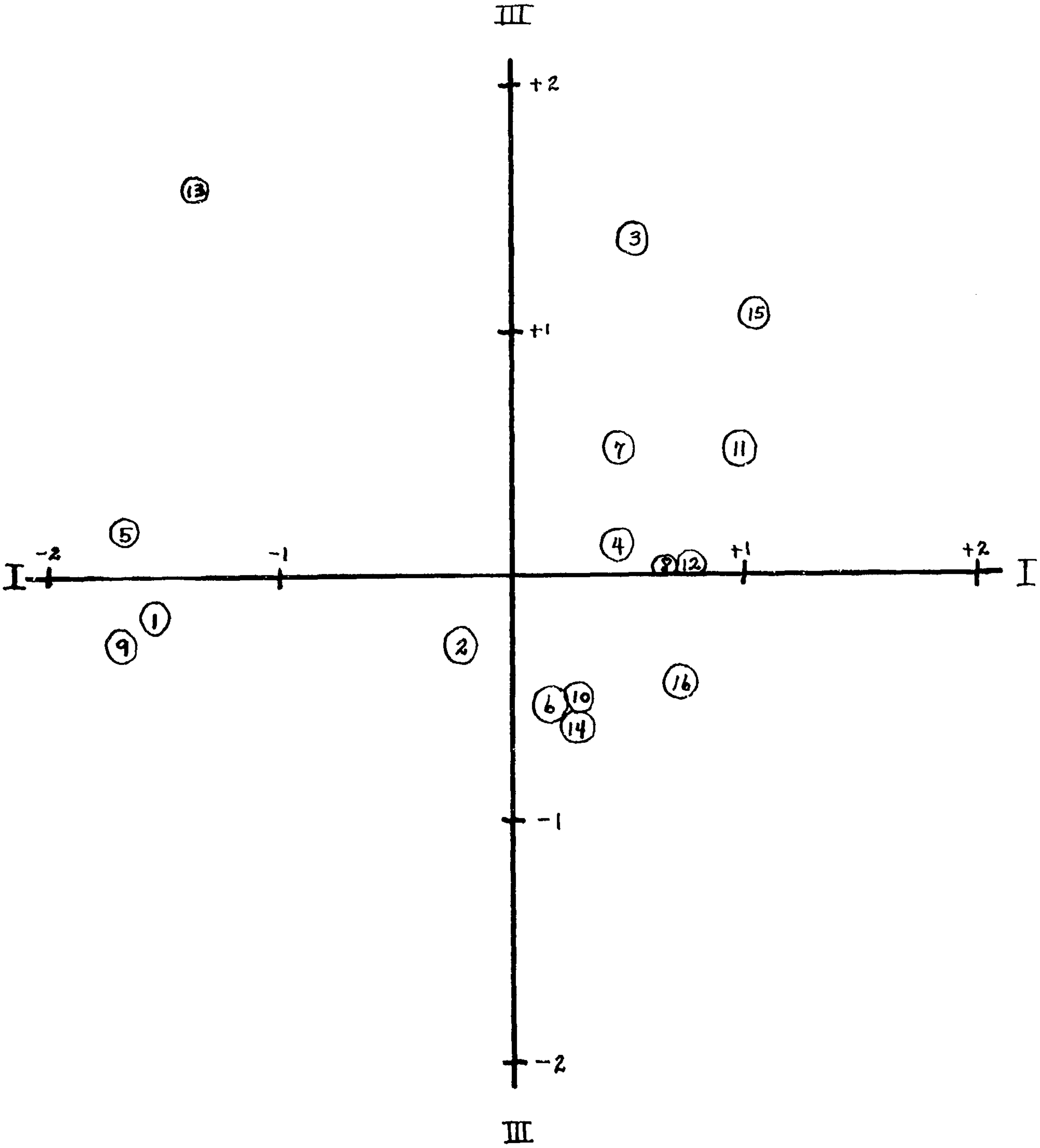

Figure 12. Factor I versus Factor III, Rotated Factor Matrix, for 16 Pure Tone Stimuli.

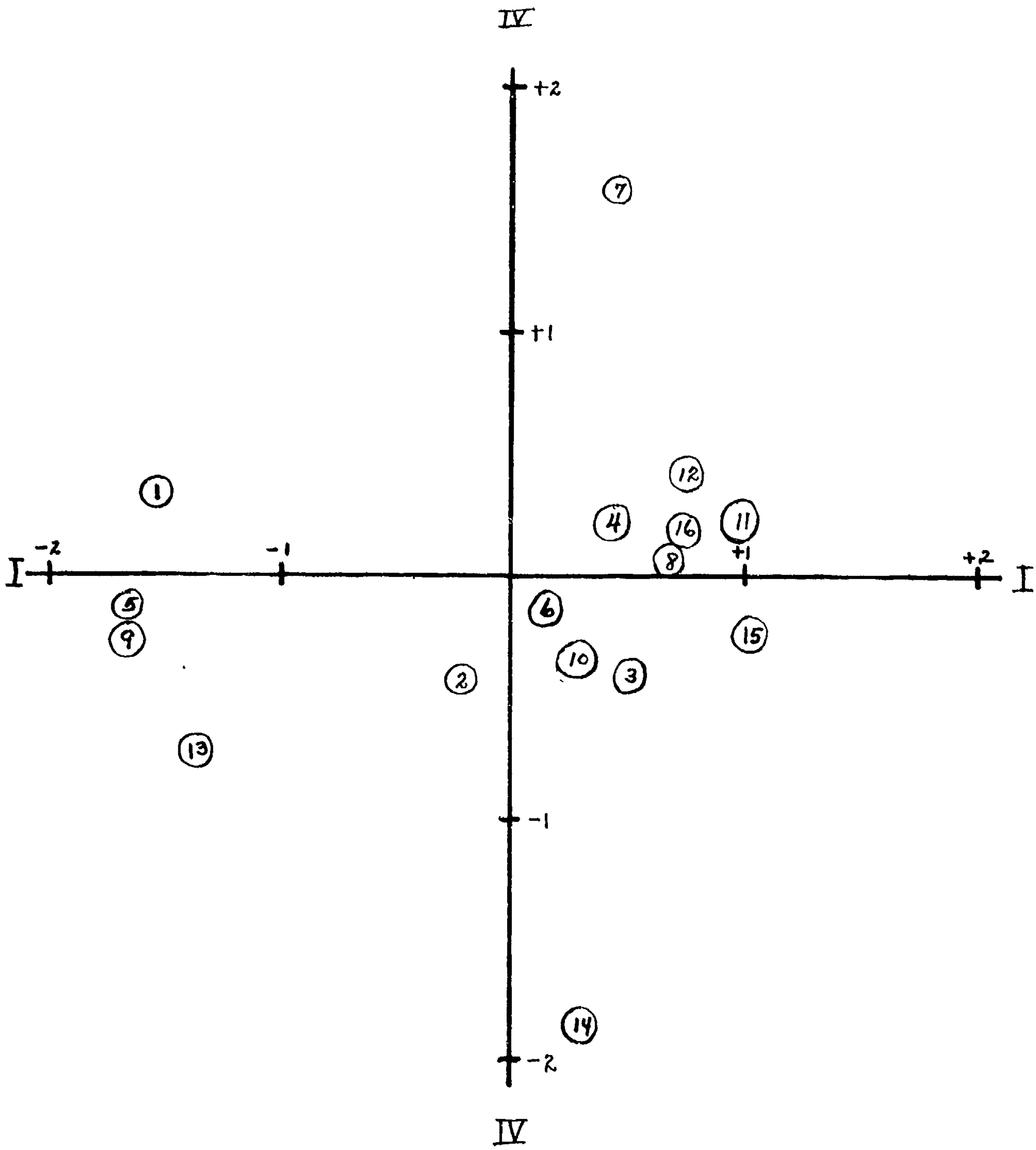

Figure 13. Factor I versus Factor IV, Rotated Factor Matrix, for 16 Pure Tone Stimuli.

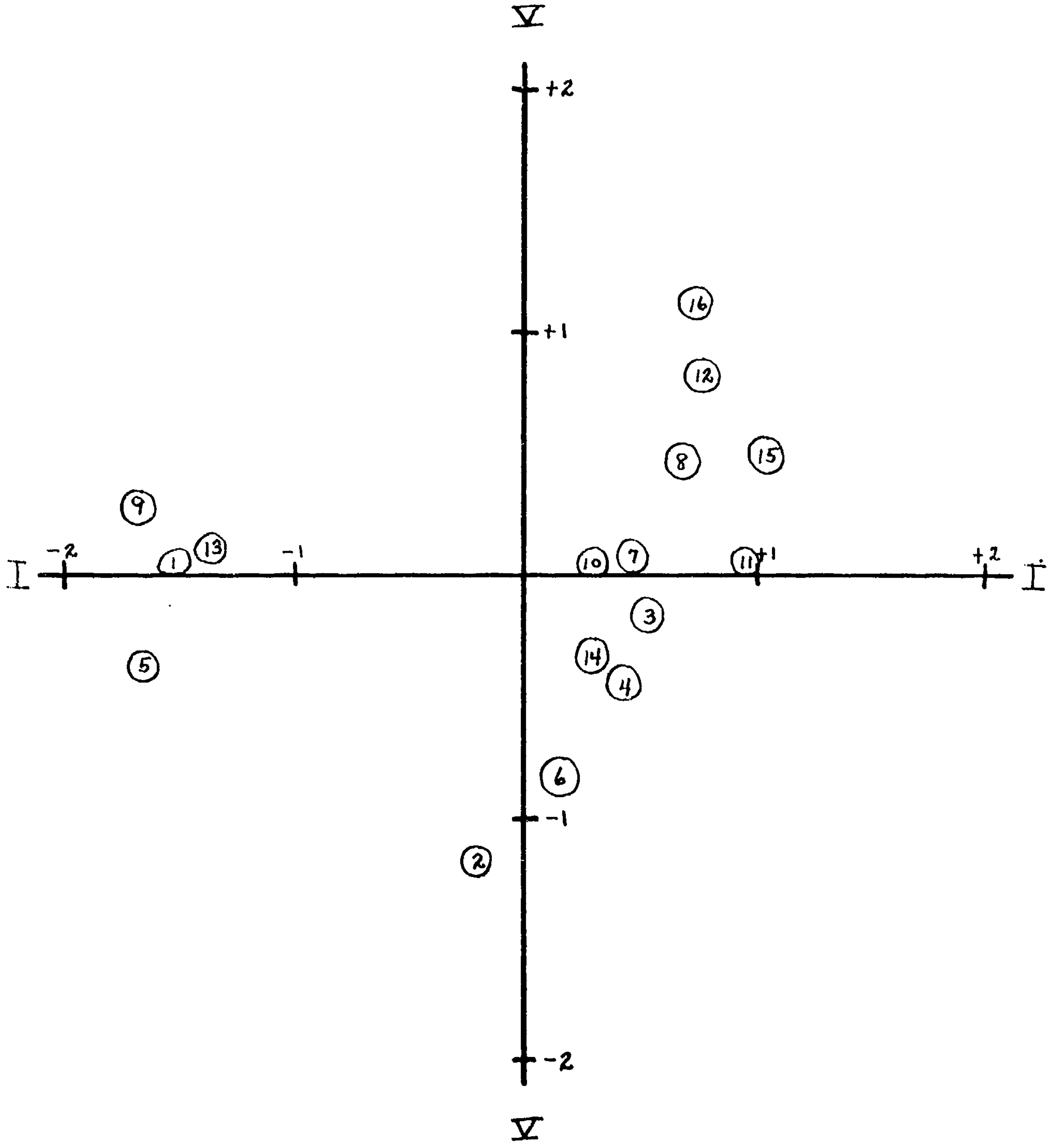

Figure 14. Factor I versus Factor V, Rotated Factor Matrix, for 16 Pure Tone Stimuli.

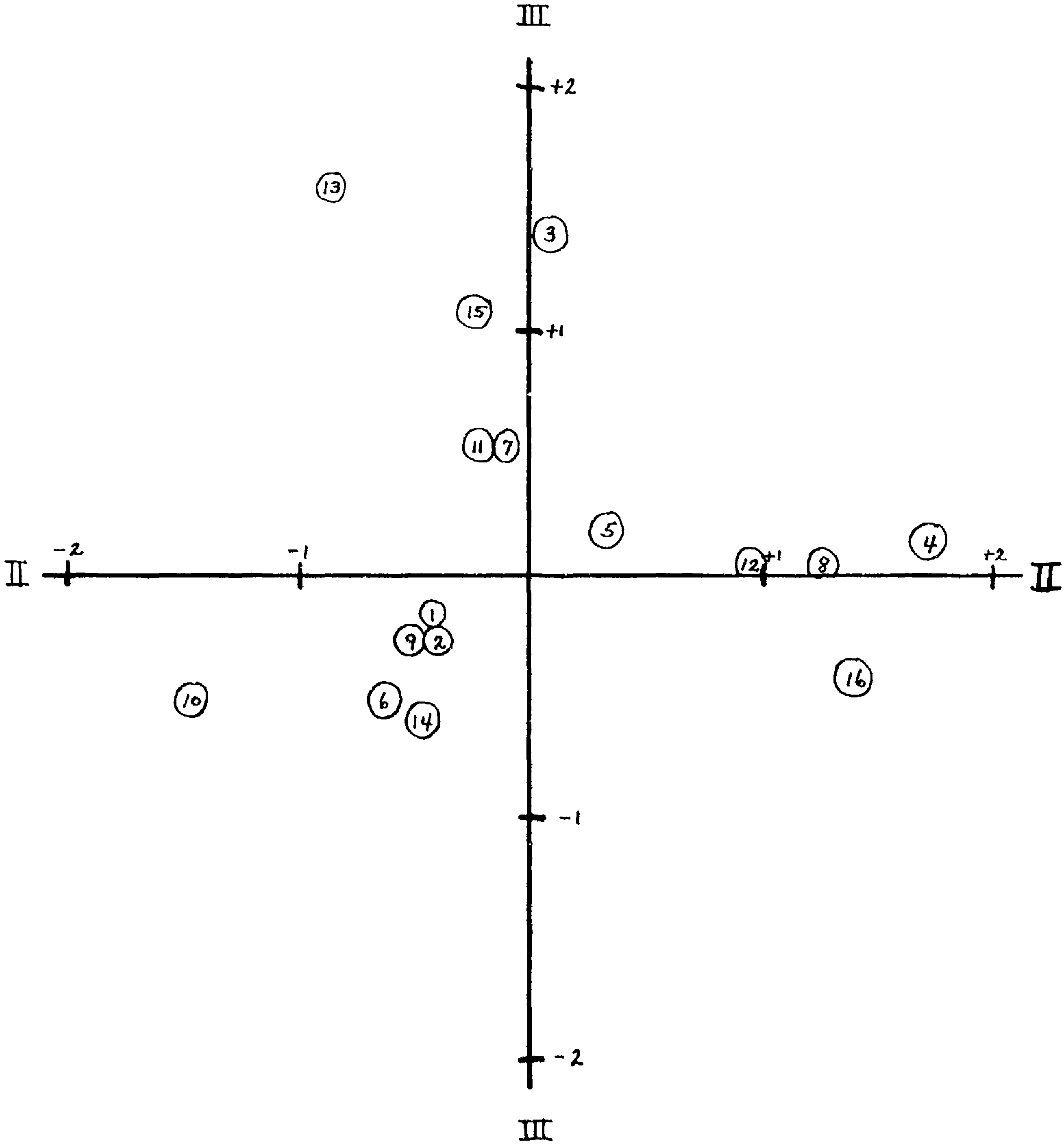

Figure 15. Factor II versus Factor III, Rotated Factor Matrix, for 16 Pure Tone Stimuli.

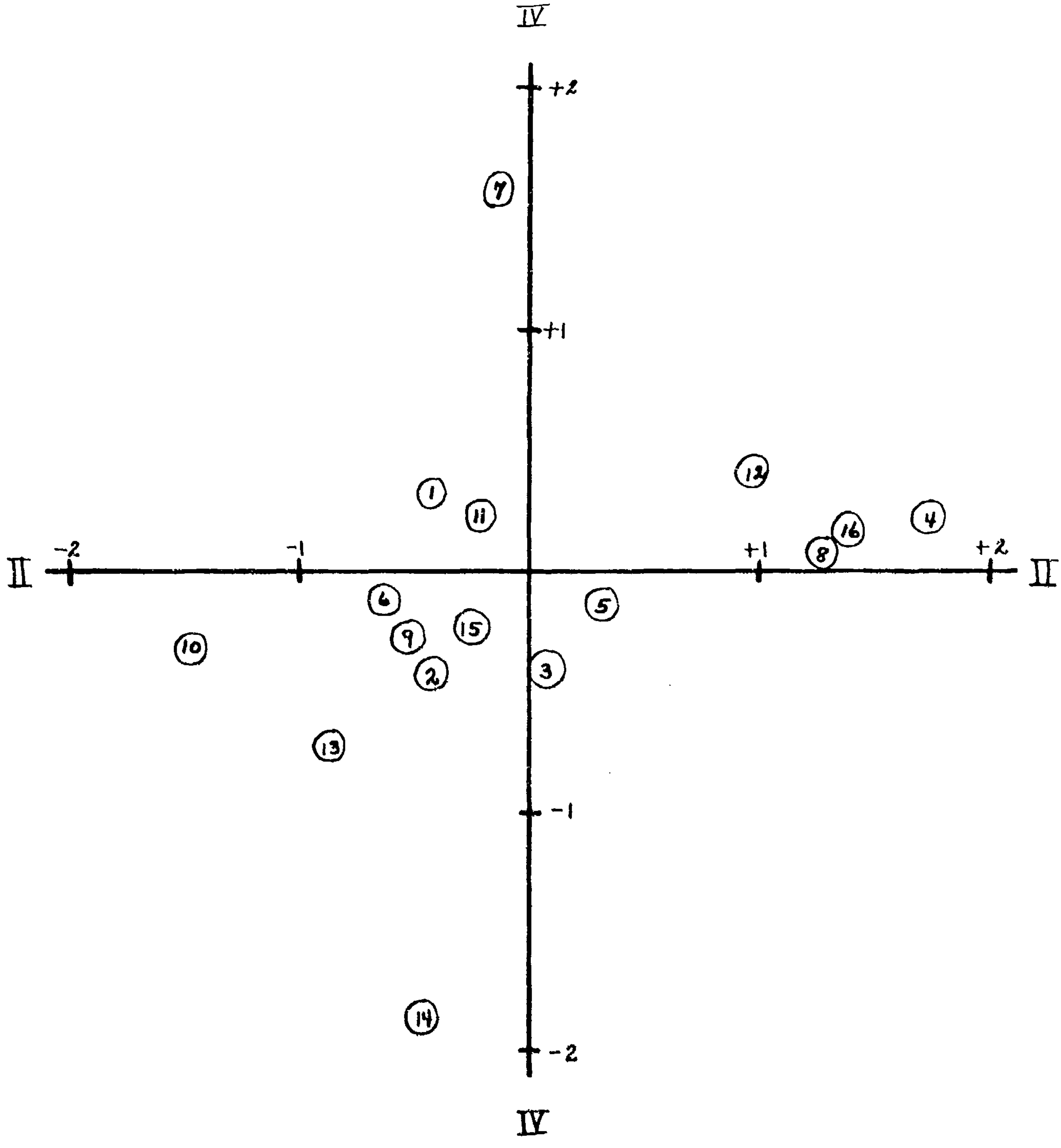

Figure 16. Factor II versus Factor IV, Rotated Factor Matrix, for 16 Pure Tone Stimuli.

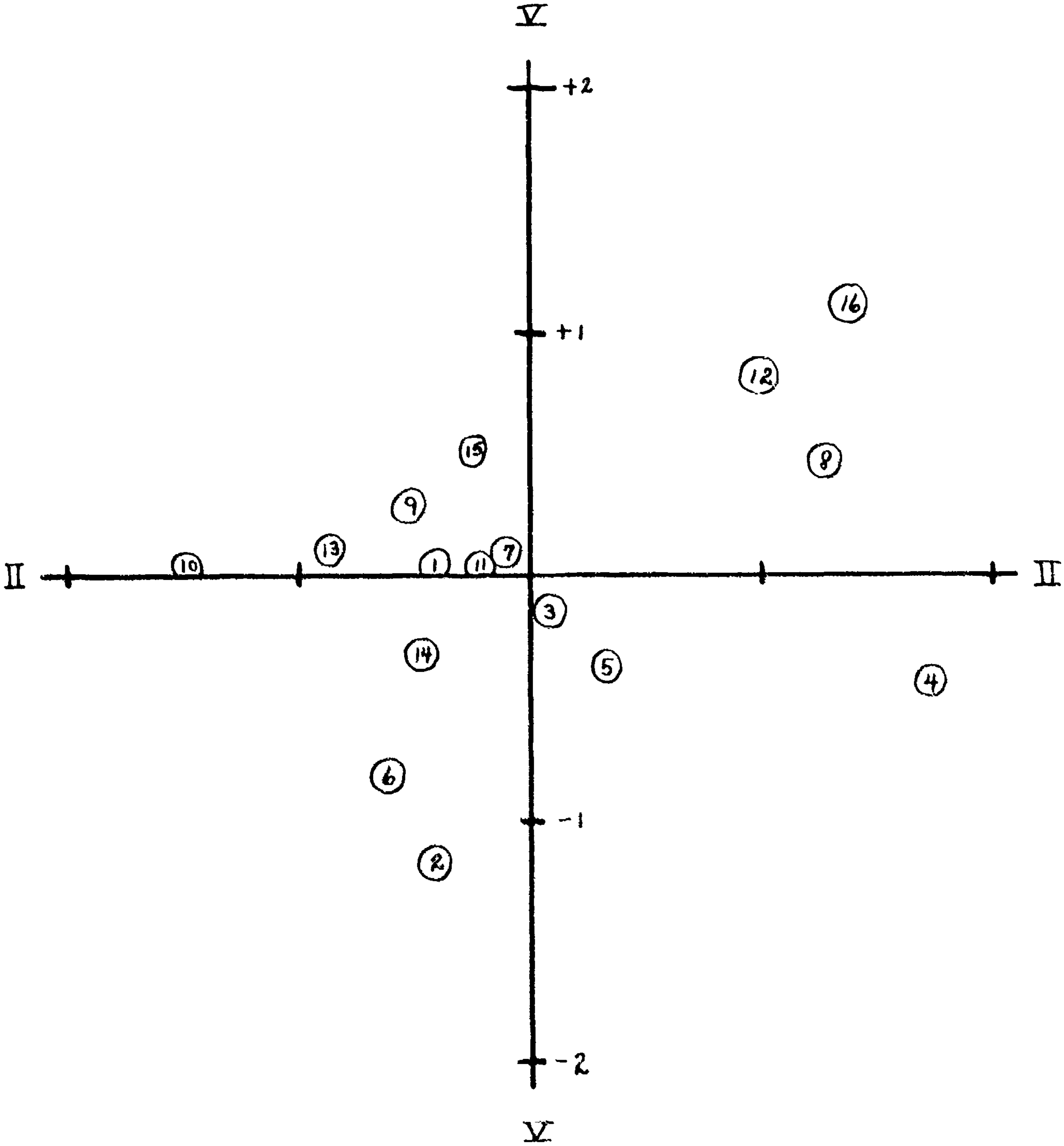

Figure 17. Factor II versus Factor V, Rotated Factor Matrix, for 16 Pure Tone Stimuli.

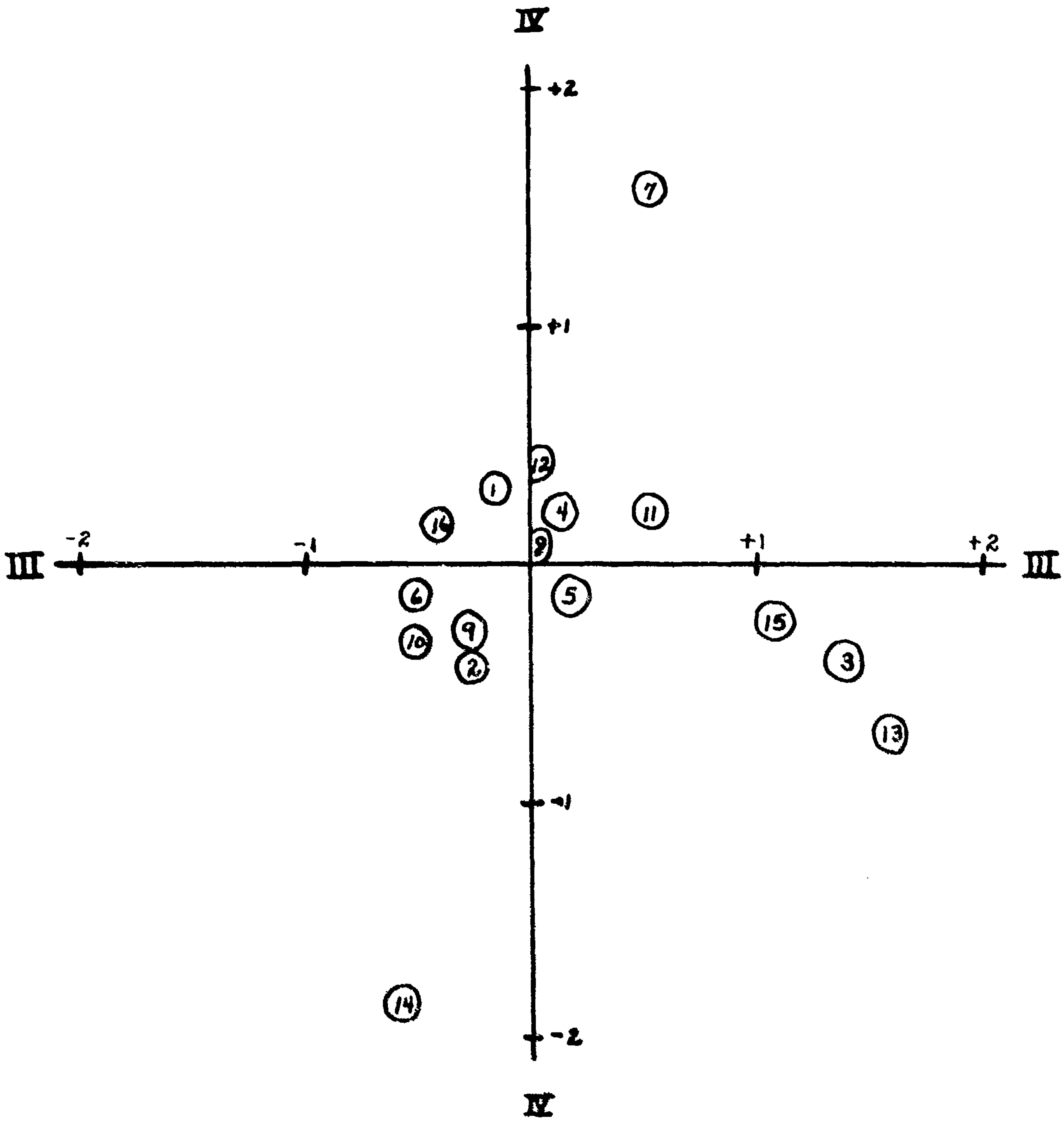

Figure 18. Factor III versus Factor IV, Rotated Factor Matrix, for 16 Pure Tone Stimuli.

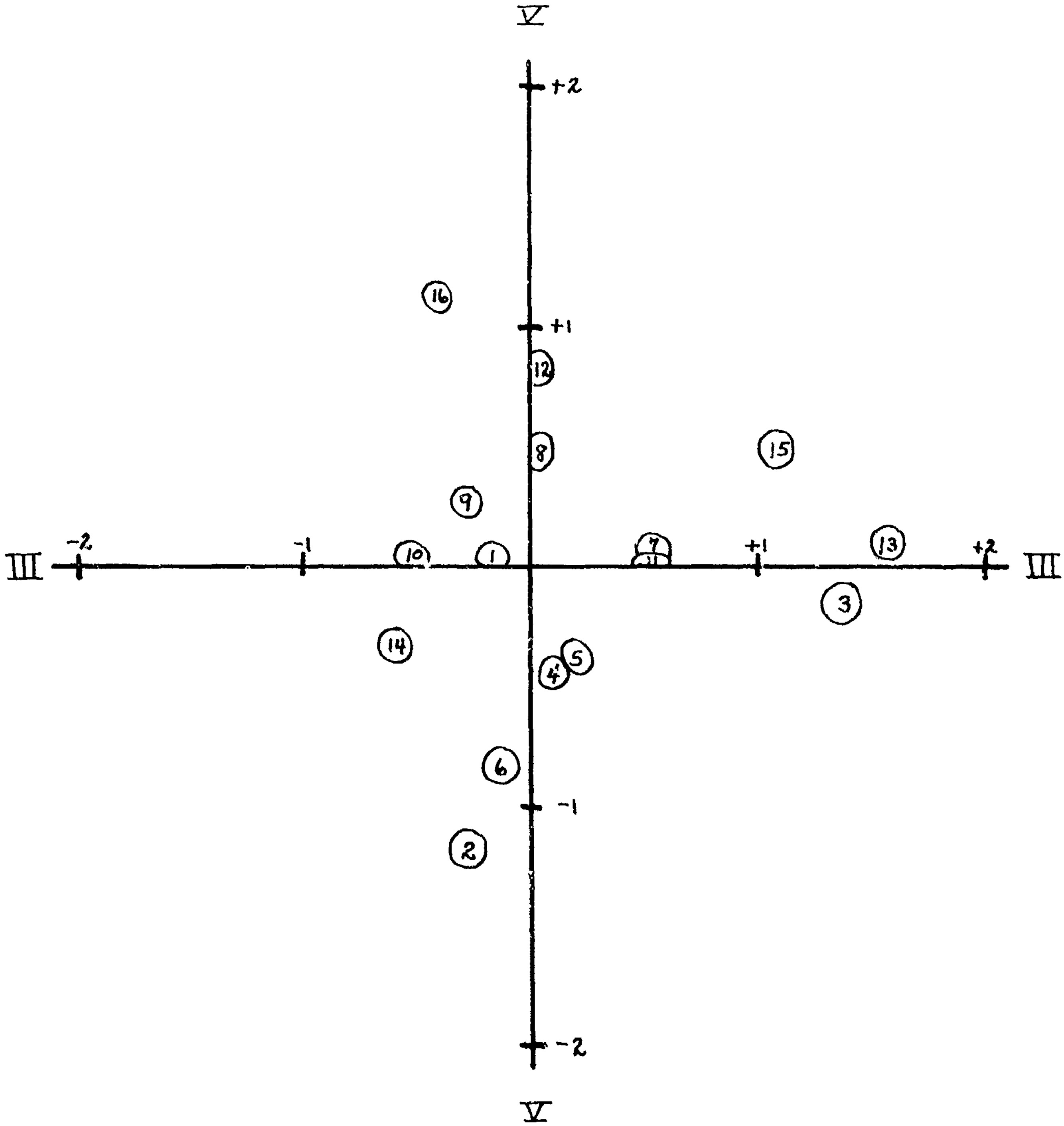

Figure 19. Factor III versus Factor V, Rotated Factor Matrix, for 16 Pure Tone Stimuli.

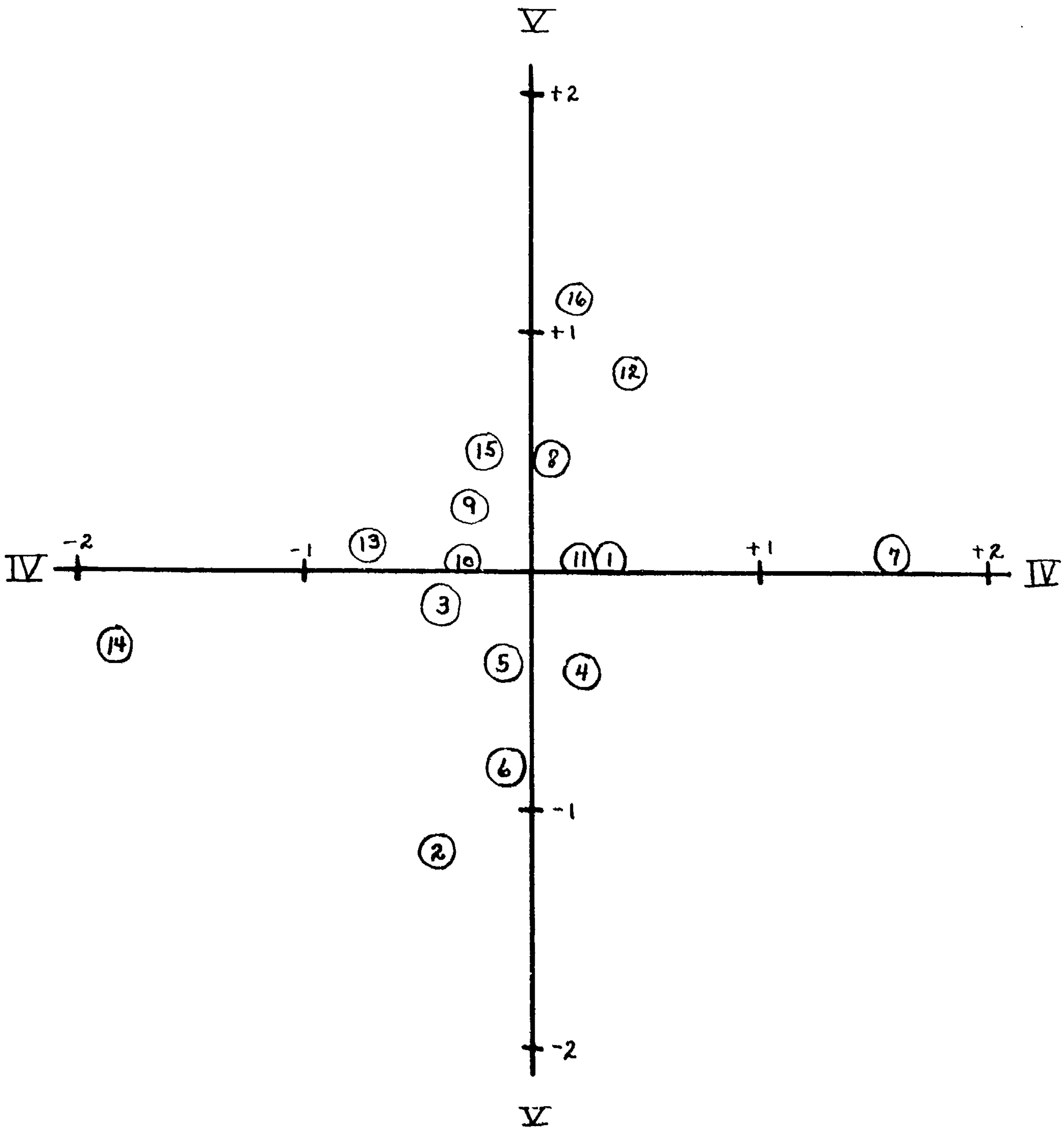

Figure 20. Factor IV versus Factor V, Rotated Factor Matrix, for 16 Pure Tone Stimuli.